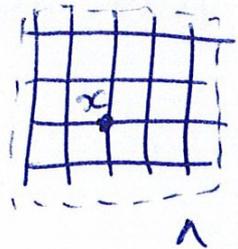


Towards  
interface fluctuations  
of the  
long-range Ising model

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# Introduction



$$\begin{aligned}\sigma_x &\in \{\pm 1\} \\ \Lambda &\in \mathbb{Z}^d \\ \Omega &= \{\pm 1\}^{\mathbb{Z}^d}\end{aligned}$$

$$\begin{aligned}H_\Lambda^\omega(\sigma) &:= - \sum_{x, y \in \Lambda} J_{xy} \sigma_x \sigma_y \\ &\quad - \sum_{\substack{x \in \Lambda \\ y \in \Lambda^c}} J_{xy} \sigma_x \omega_y\end{aligned}$$

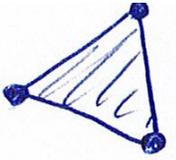
avec  $J_{xy} \geq 0$  tq  $\sum_{y \in \mathbb{Z}^d} |J_{xy}| < \infty$

$$\mu_\Lambda^\omega(\sigma) = \frac{1}{Z_\Lambda^\omega} \exp(-\beta H_\Lambda^\omega(\sigma))$$

où  $\beta > 0$

$$G_\beta = \text{conv} \left\{ \lim_{\wedge \uparrow \mathbb{Z}^d} \mu_{\wedge}^\omega \right\}$$

est un simplexe



$$\beta_c = \sup \{ \beta > 0 : \# G_\beta = 1 \}$$

def: •  $\mu \in G_\beta$  est extremale si  $\forall \nu_1, \nu_2 \in G_\beta$  tq  $\mu = \lambda \nu_1 + \nu_2^{(1-\lambda)}$   
on a  $\mu = \nu_1 = \nu_2$ .

•  $\mu \in G_\beta$  est invariante sous les translations (IT)  
si  $\forall f: \Omega \rightarrow \mathbb{R}$  locale  $\mu(f) = \mu(\theta^{-1}f)$   
 $\wedge \forall \theta: \mathbb{Z}^d \rightarrow \mathbb{Z}^d$  translation.

•  $\mu_n \Rightarrow \mu$  si  $\forall f: \Omega \rightarrow \mathbb{R}$  locale  $\mu_n(f) \rightarrow \mu(f)$ .

# Nearest neighbor interactions

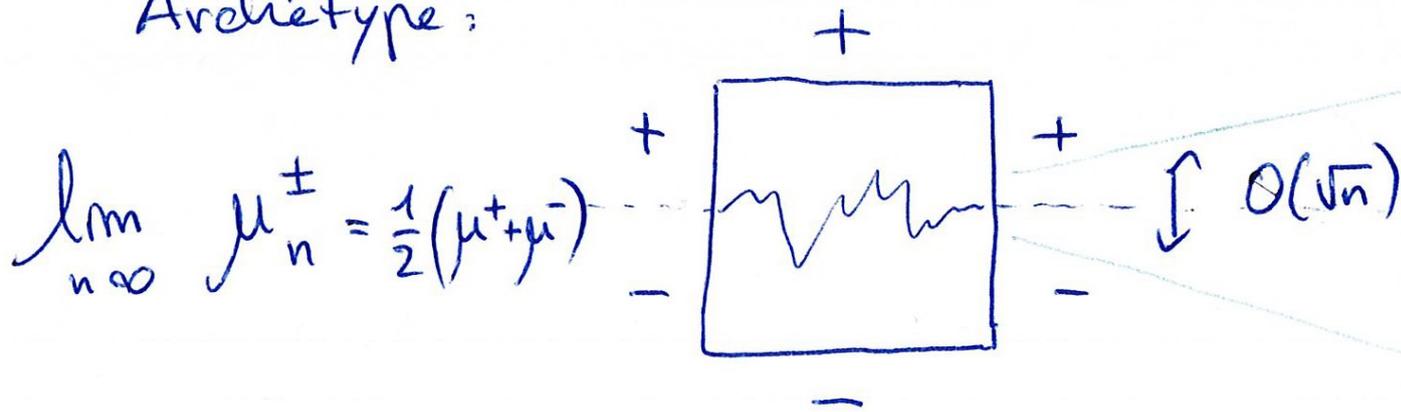
$$J_{xy} = J_{x \sim y}$$

$d=1$  pas de transition de phase =  $\beta_c = \infty$

$d=2$   $\beta_c \in (0, \infty)$   $\mathcal{G}_\beta = \{ \alpha \mu^+ + (1-\alpha) \mu^-, \alpha \in [0, 1] \}$

[AH '80'81]

Archétype:

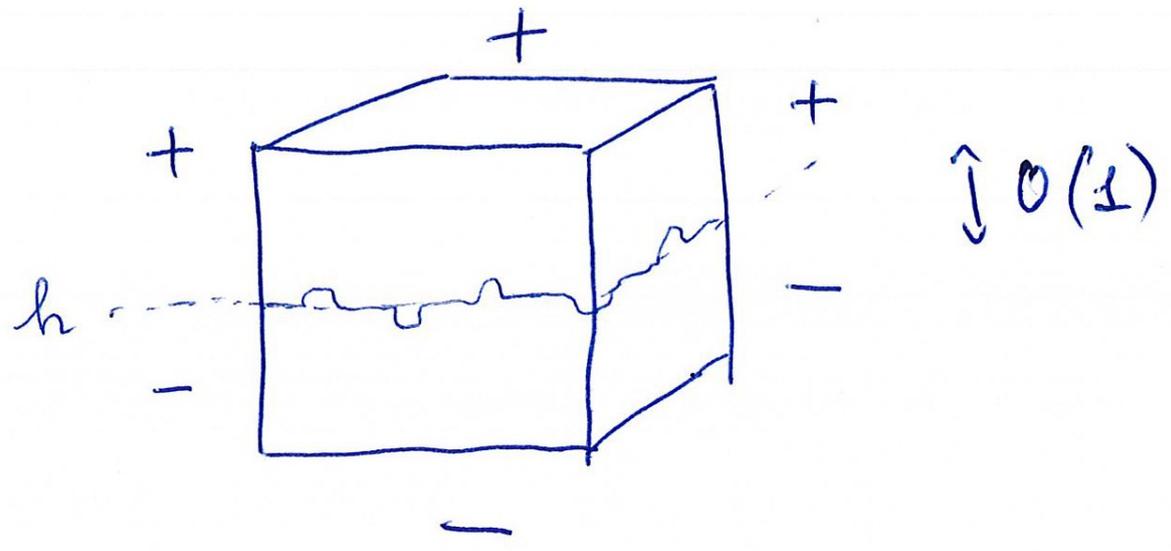


Profil de magnétisation: [AR '76]

$$\lim_{n \rightarrow \infty} \mu_n^\pm (\sigma_{(0, s\sqrt{n})}) = m^* \operatorname{sgn}(s) \Phi\left(\frac{|s|}{\sigma(\beta)}\right)$$

$d=3$

Si  $\beta \gg 1$ ,  $g_\beta$  contient au moins une sphère dénombrable de mesures extrémales non IT



=:  
États de Dobrushin

les  $\lim_{n \rightarrow \infty} \mu_n^{\pm, z}$  sont non IT  $\forall z \in \mathbb{Z}$ .

# Long-range interactions

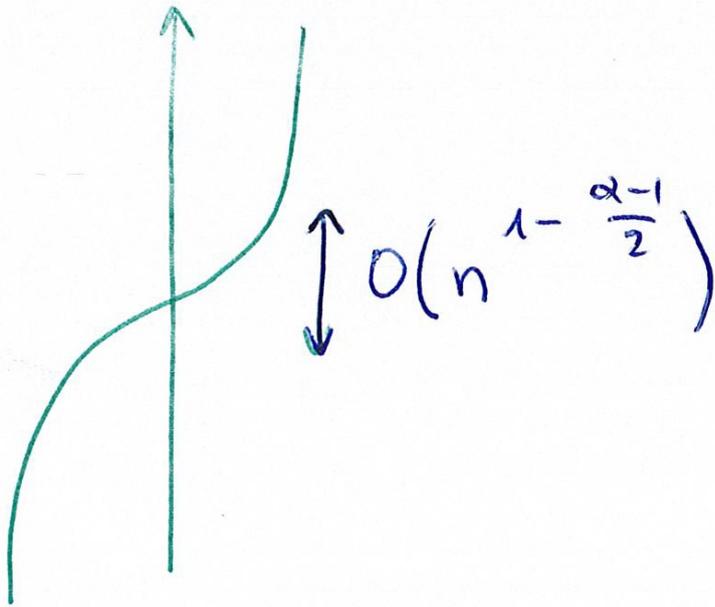
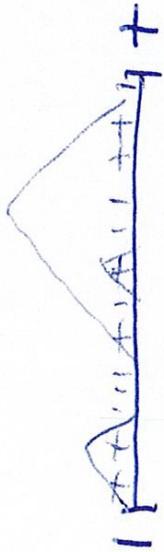
$$J_{xy} = \frac{1}{|x-y|^\alpha}$$
$$\alpha > d$$

$d=1$  : transition de phase pour  $\alpha \in (1, 2]$   
et pas pour  $\alpha > 2$ .

[Dyson '69, FS '82,] [Cassandro, Merola, Ferrari,  
 $d=2$  Presutti, Picco]

Absence d'états de Dobrushin (E non IT)

Profil de magnétisation similaire à 2d NN  
mais à une autre échelle



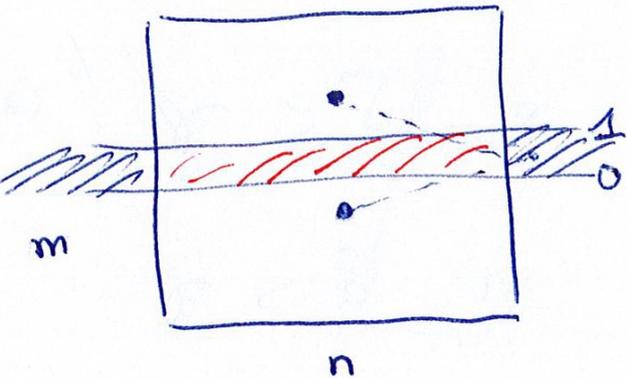
$$\lim_{n \rightarrow \infty} \mu_n^\pm \left( \sigma_{S n^{1-\frac{\alpha-1}{2}}} \right) = \text{profil de AR '76}$$

# Our theorem

with A. van Enter, A. Le Ny, W. Ruszel  
Journal of Stat. Phys. 2018

Absence of Dobrushin states  
(= extremal non-translation invariant states)  
for the 2-dimensional long-range Ising model  
for any  $\alpha > 2$  and any  $\beta > 0$ .

Preuve:



$$h(\mu_n^{\pm,0}, \mu_n^{\pm,1})$$

$$= \left\langle \frac{\mu^0}{\mu^1} \log \frac{\mu^0}{\mu^1} \right\rangle_n^1$$

$$\stackrel{\text{strip}}{=} \left\langle H_n^0 - H_n^1 \right\rangle_n^1 + o(1)$$

$$= - \sum_{x \in \Lambda} \sum_{y \in \text{strip}} |x-y|^{-\alpha} \langle \sigma_x \rangle_n^0$$

$$\stackrel{\text{sym ferro}}{\leq} \sum_{x \in \text{strip}} \sum_{y \in \text{strip}} |x-y|^{-\alpha}$$

$$= O(n^{2-\alpha})$$

$< \infty$  pour  $d > 2$ .

$\Rightarrow \mu^{\pm, 0}$  et  $\mu^{\pm, 1}$  sont mut. abs. cont.

$\Rightarrow$  elles ont les mêmes mesures dans leur déc. extrêm.

$\Rightarrow$  si  $\mu^{\pm, 0}$  extrémale alors  $\mu^{\pm, 0} = \mu^{\pm, 1}$

invariante par translations

□

# Ongoing work with R. Durand and W. Ruszel

Conjecture: cf [FZ] pour la chaîne gaussienne discrète

$$h(\mu_n^{\pm, 0}, \mu_n^{\pm, n^\delta}) \rightarrow \begin{cases} < \infty & \text{si } \delta < \gamma \\ \infty & \text{si } \delta > \gamma \end{cases}$$

où  $\gamma = \min \left\{ \frac{\alpha - 2}{2}, \frac{1}{2} \right\}$ .

